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EUROPEAN
SCHOOLS
1993

6th/7th Year
MATHEMATICS
SYLLABUS
3 period course

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3-period course
ANALYSIS

SUBJECTS	KNOWLEDGE & SKILLS	POSSIBLE TEACHING APPROACHES
1) Series Arithmetic Progressions Geometric Progressions	*Pupils must be able to:* - define an Arithmetic Progression (A.P.) - recognise an A.P. and determine the first term and common difference . - give the general expression of an A.P. in terms of n. - calculate the sum of consecutive terms in an A.P.. - calculate the sum of the first n positive integers. - apply the theory of A.P.'s to problems. - define a Geometric Progression (G.P.) - recognise a G.P. and determine the first term and common ratio. - give the general expression of a G.P. in terms of n . - calculate the sum of consecutive terms of a G.P. . - establish the formulas: $1 + q + q^2 + q^3 + \ldots\ldots + q^n = \frac{1 - q^{n+1}}{1 - q}$ $(1 - q)(1 + q + q^2 + q^3 + \ldots\ldots + q^n) = 1 - q^{n+1}$ - apply the theory of G.P.'s to problems.	Introduce the idea of the series of the type: $u_{n+1} = f(u_n)$ e.g. Simple Interest etc. e.g. Compound Interest, growth phenomena etc.

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2) Functions	*Pupils must be able to:* - define a function and give examples of functions. - define, apply and study the following ideas; domain of definition; the image of f on a closed interval {a,b}; the graph of a function - use the notation : $f: A \to B; A, B \subset R$ $x \to y = f(x)$ - draw, using scales,the graphs (curves) of functions defined in rectangular cartesian co-ordinates: $f(x) = ax^2 + bx + c$ $f(x) = ax^3 + bx^2 + cx + d$ $f(x) = \frac{ax + b}{cx + d}$ $a, b, c, d \in R$ (continued...)	Extend work already covered in 4th and 5th Years.

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2) Functions (continued)	*Pupils must be able to:*	
	- determine algebraically or graphically : ✓ the zeros (roots) of a function; the points of intersection of curves.	Continue with the same type of functions as solved in 4th & 5th years.
	- study the symmetry of functions and use symmetry arguments to draw curves. $f(-x) = f(x)$; $f(-x) = -f(-x)$	
	- understand the geometry of increasing and decreasing functions.	
	- determine numerically the limits of $f(x)$ as x tends towards $\pm\infty$	An opportunity to make use of calculators. A theoretical approach is not advised.
	- use functions to solve problems and to associate functions with problems.	Examples could include free fall, projectiles, etc.

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	Pupils must be able to:	
3) Introduction to derivitives	- recognise the geometric properties of a chord, a tangent and a circle. - establish the equation of a chord for the curve of a function and show that the gradient of the chord is given by: $m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\Delta f}{\Delta x}$	
	- construct a tangent at a point on a curve. ✓ - define a tangent at a point on a curve. ✓ - calculate the gradient of a tangent . ✓ - define a normal at a point on a curve. ✓ - calculate the gradient of a normal. ✓	Tangents showing velocities and acceleration.
✓ **4)** The derivitive function f ' (x)	- understand the concepts of a derived value at a point and a derivitive function. - establish the equation of a tangent and a normal at a point on a curve. - find derived functions for all the functions given in section **2)** .	Limit this study to polynomial functions of degree 3 or less and reciprocal functions of the type described in section **2)**

(continued...)

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4) The derivitive function f '(x) (continued.)	*Pupils must be able to:* -use the following differentiation rules: $(f+g)'(x)$; $(a.f)'(x)$ $(f.g)'(x)$ $\left(\frac{f}{g}\right)'(x)$ $[f(ax+b)]'$ p 324 - apply these rules for all the functions given in section **2)**. - study the variations of functions with the help of derived functions. - study the stationary points (extrema) of a function and determine the co-ordinates of any maximum and minimum points. ✓ (continued...)	Teachers can choose any appropriate method

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	Pupils must be able to:	
4) The derivitive function f '(x) (continued....)	- study functions completely (domain of definition, zeros, extrema, increasing and decreasing intervals). - recognise and apply derivitives to problems in topics other than mathematics such as velocity and cost of living. - solve simple problems of maximising and minimising.	Indicate the significance of the second derivitive
5) Circular Functions	- plot curves of the type: $f(x) = \sin x$ (x in radians) $f(x) = \cos x$	Introduce the notion of waves
	- determine the domain, the zeros, the period and the graph of the functions: $f(x) = \sin(ax + b)$ $f(x) = \cos(ax + b)$	
	(continued.....)	

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	Pupils must be able to:	
5) Circular functions (continued..)	- apply the derivitives of f (x) = sinx and f (x) = cosx. - study the functions of the type: $f(x) = m + k.\sin(ax + b)$ $f(x) = m + k.\cos(ax + b)$ $m, k, a, b \varepsilon R$	
6) Integration	- give the definition of a primitive p 460 - establish primitives for the following functions: $f(x) = a$ $f(x) = x^m$ $m>1$ or $m=1$ $f(x) = \sin x$ $f(x) = \cos x$ (continued...)	

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6) Integration (continued....)	*Pupils must be able to:* - show , using simple examples, that the area enclosed by a curve,the x-axis and two lines x = a and x = b (where b>a) can be calculated using primitives of functions. - define and interpret: $\int_a^b f(x)\,.dx = F(b) - F(a)$ - apply the formulas: $\int_a^b f(x).dx + \int_b^c f(x).dx = \int_a^c f(x).dx$ $\int_a^b f(x).dx = -\int_b^a f(x).dx$ $\int_a^a f(x).dx = 0$ (continued)	Use the example f (x) = ax + b or numerical methods.

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6) Integration (continued.....)	*Pupils must be able to:* - deal with the following rules of integration: $\int_a^b (f+g)(x).dx$ $\int_a^b k.f(x).dx$ $\int_a^b f(\alpha x+\beta).dx$ $\alpha,\beta,k \in R$	Teachers could introduce the method of substitution.
	- apply integral calculus to calculating areas enclosed by a curve and the x-axis or by two curves or by a curve, the x-axis and the lines x = a and x = b ;	Problems should be restricted to those where the determination of intersection points is algebraically simple. Show that the integral calculus can be used to calculate energies, average value etc.

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		Pupils must be able to:	
7) Exponential Functions Logarithmic Functions	×	- describe exponential growth using the function $f(x) = b^x$ $(b>1)$	Use calculators and interpret graphs. Avoid proofs.
	×	- sketch (draw) curves of the functions $f(x) = b^x$ (for $b>0$) in rectangular cartesian co-ordinates.	
	×	- use numerical techniqes to find limits as $x \to \pm\infty$, recognise the different curves for different values of b and be aware that all the curves pass through the same point.	
		- sketch (draw) the curves of the functions $f(x) = ab^x$ and apply these functions to solve problems.	
	×	- define a logarithm in a base b and study the function	
	×	$f(x) = \log_b(x)$ where $b>0$ and $b \neq 1$.	
		- sketch (draw) curves of logarithmic functions.	
		- use numerical techniques to find the limits as $x \to \pm\infty$	Link up intuitively with reciprocal functions.
	×	and as $x \to 0^+$, recognise the different curves for different values of b and be aware that all the curves pass through the same point.	
		- see the link between $f(x) = b^x$ and $f(x) = \log_b(x)$	

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8) Functions e^x and Ln x	*Pupils must be able to:* - define the number e and the functions $f(x) = e^x$ and $f(x) = \text{Ln } x$. Sketch (draw) the curves of these functions. - use the rules of algebra and arithmetic with regard to e^x and Ln x. - differerentiate the functions $f(x) = e^x$ and $f(x) = \text{Ln } x$. - study the variations of the functions of the type: $f(x) = e^{ax+b}$ $f(x) = \text{Ln}(ax+b)$ $f(x) = (ax+b)e^{cx}$ $a,b,c \in R$ - calculate integrals of the type: $\int_{\alpha}^{\beta} e^{ax+b}.dx$ and $\int_{\alpha}^{\beta} \frac{ax+b}{cx+d}.dx$ - apply these functions to problems.	Teaching method at the discretion of teachers. Establish the link between $f(x) = e^x$ and $f(x) = \text{Ln } x$ Avoid unecessary theory or proof. Use the reduced form: $\frac{ax+b}{cx+d} = A + \frac{B}{cx+d}$ Deal with problems on growth previously used.

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	Pupils must be able to:	
9) Probability Combinations Permutations	- appreciate the idea of probability of an event A. - use the formulas: $p(\sim A) = 1 - p(A)$ $p(A \cap B) = p(A).p(B)$ (for independent events) $p(A \cup B) = p(A) + p(B) - p(A \cap B)$	This section is a revision of work done in 4^{th} and 5^{th} years.
10) Statistics	- determine the frequency of discrete data (scores) - tabulate scores. - calculate the arithmetic mean μ of a set of scores. - calculate the standard deviation σ of a set of scores. - represent the frequency of scores graphically. - use any calculations made to describe the distribution. (continued...)	Introduce intervals and mid-interval values. Make extensive use of calculators.